ENTRAINMENT, DISPLACEMENT AND TRANSPORT OF TRACER GRAVELS

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ABSTRACT

The mass and size distribution of grain entrainment per unit bed area may be measured by replacing a volume of the bed with tracer gravels and observing the mass difference before and after a transport event. This measure of spatial entrainment is relevant to any process involving size-selective exchange of sediment between transport and bed and may be directly used in calculations of sediment transport rate using an elementary relation for fractional transport components presented here. This relation provides a basis for evaluating tracer data collected by different methods and may be used to provide physical insight regarding the expected behaviour of tracer grains. The variation with grain size of total displacement length \( L_{ti} \) depends on the degree of mobilization of the individual fractions on the bed surface: \( L_{ti} \) is independent of \( D_i \) for smaller, fully mobile sizes and decreases rapidly with \( D_i \) for larger fractions in a state of partial transport (in which a portion of the surface grains remain immobile through the flow event). The boundary between fully and partially mobile grain sizes increases with flow strength. These inferences are supported by values of \( L_{ti} \) calculated from flume experiments and provide a physical explanation for a summary relation between \( L_{ti} \) and \( D_i \) based on field data.

INTRODUCTION

Sediment movement in gravel-bed rivers can be measured by direct sampling of the transport rate or by using tracer gravels to determine entrainment rates and grain displacement lengths. Although direct sampling is most common, tracer gravels offer some particular advantages and have found increased use in recent years (e.g. Emmett et al., 1990; Church and Hassan, 1992; Hassan et al., 1992; Schmidt and Ergenzinger, 1992; Haschenburger, 1996). Tracers are well suited to the stochastic and spatially variable nature of bedload transport because they are based on a predetermined bed sample composed of individual grains. For example, the entrained proportion of the bed surface and the size distribution of mobile and immobile grains may be measured with tracers, but not with direct sampling of sediment transport rate. Tracers also provide logistical and safety advantages because they may be installed during low flow, thereby avoiding direct sampling of bedload during floods.

The development of tagging methods that permit recovery of a large proportion of moved tracers (Ergenzinger and Conrady, 1982; Hassan et al., 1984) has increased the accuracy of determining displacement distance and, therefore, transport rates from tracer observations. Tracers are likely to provide a more accurate measure of small transport rates, for which the errors associated with bedload samplers may exceed the measured rates. Where the accuracy of measuring transport rates with tracer gravels is comparable to that possible with direct sampling, the logistical advantage and additional information provided by tracer gravels make them an attractive alternative to direct sampling.
This paper addresses several aspects of the collection, analysis and interpretation of tracer gravel observations, with the general objectives of increasing the information yield of a tracer gravel programme and improving the accuracy of transport estimates from tracer gravels. First, a method is described for using tracer gravels to accurately measure the total mass and the size distribution of sediment entrained per unit bed area. Direct measurements of these quantities are immediately relevant to mechanisms of sediment exchange between the transport, bed surface and bed subsurface, and supplement work on the vertical mixing of tracer grains (e.g. Schick et al., 1987; Hassan and Church, 1994). The accuracy of bedload calculations depends equally on the quality of entrainment and displacement observations, so the method presented here complements the recent advances in tracer recovery and displacement estimates. In practice, tracer measurement of sediment entrained per unit bed area provides a useful complement to scour chains and other scour depth indicators and should give a superior measurement of entrainment at small transport rates with negligible bed scour.

The second objective of the paper is to present an elementary relation between transport rate, displacement length, and the spatial entrainment of different grain sizes. An explicit statement of the relation between transport rate and its components is needed not only to calculate transport rate, but also to compare different types of tracer measurements, and to evaluate tracer observations from different flows and locations. The transport component relation distinguishes between terms that have time-invariant average or limiting values and terms that vary directly with the duration and rate of transport. The former are suitable for direct comparisons among different flow strengths and sediments; unless suitably scaled, comparison of the latter results from stream to stream is not particularly meaningful, because they become arbitrarily large or small depending on the transport duration.

The final objective of the paper is to use the explicit relation for transport components to illustrate the functional dependence of the individual components, thereby providing a physical basis for evaluating field observations. The expected variation with grain size of tracer displacement length is developed and illustrated using displacement lengths calculated from measurements of entrainment, surface size distribution and transport rate in laboratory experiments. These results are compared to a summary relation for displacement lengths observed in the field (Church and Hassan, 1992), leading to a suggested physical mechanism for the observed relation.

ENTRAINMENT FROM LARGE TRACER GRAVEL INSTALLATIONS

If all grains in an area of the bed are replaced by tracer gravels to a depth greater than the scour depth, comparison of the mass of tracers before and after a flow event provides a direct measure of the sediment entrainment per unit bed area \( M_a \). The product of \( M_a \) and the total displacement length of mobilized tracers is the mean bed material transport rate. Such a tracer gravel installation also provides a local measure of the proportion of the bed surface mobilized and the size distribution of mobilized grains.

For installation, the tracer gravel method presented here consists of little more than marking and replacing all the grains that would be collected in a large sample of the bed material size distribution. A large metal cylinder (e.g. a cut-off oil drum) is worked into the bed as deeply as possible (similar to the method of McNeil and Ahnell (1964) but with a larger sampler). Use of a cylinder provides a well-defined sample volume and permits both subaerial and underwater sampling, as long as the water depth is shallow enough to permit access to the cylinder interior (roughly less than 0.5 m). All sediment down to the bottom of the sampler may be removed manually or by using a freeze-core device within the cylinder (Rood and Church, 1994). The size of cylinder should be selected to provide a sample of sufficient size to give an accurate measure of the bed size distribution. We have used cylinders of 60 m diameter and recovered samples as large as 280 kg by removing sediment to a depth of 45 cm. The depth of the sample is measured by surveying the bed before and after removing the sample.

After the size distribution of the sampled sediment is determined, the sediment is painted and returned to the sampler and the cylinder is removed from the bed. Complete recovery of all immobile tracers can be facilitated if only a portion of the bed sample is marked and returned to the centre of the sample volume using a smaller cylinder placed within the first cylinder. The disturbance produced by removing and replacing sediment may be partly mitigated by dividing the sample into vertical subsamples which are then returned to the bed in reverse
order. Accurate vertical subsamples may be manually collected for the subaerial case. For samples in the wetted portion of the streambed, vertical segregation of the fine fractions is difficult to maintain with manual sampling and the combined cylinder–freeze-core technique of Rood and Church (1994) is a useful alternative.

This tracer method requires considerable labour. For gravel and cobble beds, the size of a representative sample can be several hundred kilograms (Church et al., 1987; Rood and Church, 1994). We find that one or two samples can be installed in a day by a team of two people (Wilcock et al., 1996). The fieldwork may be expedited if premarked tracer grains are used, thereby saving the time required to dry and paint the sample. In this case, the native gravel is replaced by marked grains matching the number and shape of grains of each size. Grains larger than 8 to 16 mm may be replaced by the number and shape of grains in each size class. Sand and granule sizes are conveniently replaced on a total mass basis using brightly painted sediment such as may be found commercially for aquarium use. The effort involved in installing large tracer gravel samples is balanced by the fact that such samples are necessary to determine the size distribution of the bed so that, in many cases, much of the sampling effort may already be part of the field programme.

After the flow event, the tracers are resampled to determine the mass of grains of each size remaining in place, from which the total entrained mass \( M_e \) and the entrained mass of each size \( M_i \) may be calculated. Dividing by the sample area gives the mass entrainment per area on a total (\( M_a \)) or size-fraction (\( M_{a_i} \)) basis. The total depth of bed scour, or exchange depth \( d_e \), is calculated as \( d_e = (M_a / M_i) d_s \), where \( M_i \) is the initial mass of tracer grains, \( (M_a / M_i) \) is the proportion of entrained grains, and \( d_s \) is the surveyed sample depth. A fractional exchange depth \( d_{x,i} \) may be calculated as the entrained proportion of grains of each size multiplied by the sample depth. To provide a consistent comparison among samples with different grain size and sample depth, the exchange depth may be scaled by the thickness of the bed surface layer, for which \( D_{90} \) provides an appropriate estimate. The scaled exchange depth is:

\[
\frac{d_{x,i}}{D_{90}} = \frac{M_{x,i}}{M_i} \frac{d_{x,i}}{D_{90}}
\]

Because \( M_{x,i} / D_{90} \) approximates the mass of grains in the surface layer, \( d_{x,i} / D_{90} \) may be interpreted as the exchange depth expressed in multiples of the surface layer thickness. For \( d_{x,i} / D_{90} < 0.4 \), Equation 1 provides an estimate of the mobilized proportion of the bed surface. A similar interpretation can be given to the scaled fractional exchange depth \( d_{x,i} / D_{90} \). Values of \( d_{x,i} / D_{90} < 1 \) indicate a state of partial transport, in which only a portion of the surface grains of a given size are mobilized over the duration of the transport event (Wilcock and Mc Ardell, 1997).

An example of the information provided by large tracer gravel installations is given in Figure 1, in which the scaled fractional exchange depth \( d_{x,i} / D_{90} \) is plotted as a function of total exchange depth \( d_{x,i} / D_{90} \) for two reaches on the Trinity River, California (Wilcock et al., 1996). The data plotted represent the mean of three samples at each of three locations for two reservoir releases of different magnitude. Two groups of samples were collected in both reaches at the lower flow, and one group of samples in both reaches at the higher flow. The samples were collected manually in a water depth of 0.5 m or less.

The fractional exchange depths broadly follow the total exchange depth, although with some size-dependent variation. For very little entrainment (\( d_{x,i} / D_{90} \leq 0.1 \)), the finest sizes at both sites show weak partial transport (0.2 < \( d_{x,i} / D_{90} \) < 0.4), whereas all coarser sizes are essentially immobile. For \( d_{x,i} / D_{90} = 0.4 \) and all sizes finer than \( D_{98} \), some grains of each size are moved although all sizes are in a state of partial transport (\( d_{x,i} / D_{90} < 1 \)), with the exception of \( D_{66} \) at Poker Bar, an outlier that appears not to be representative of the local entrainment. For complete surface entrainment (\( d_{x,i} / D_{90} = 1 \)) at the Steelbridge site, sizes larger than 90 mm (=\( D_{72} \)) are in a state of partial transport, whereas \( d_{x,i} / D_{90} \geq 1 \) for all finer sizes. For \( d_{x,i} / D_{90} = 1.4 \) at the Poker Bar site, the surface portion of sizes larger than 32 mm (=\( D_{90} \)) is completely mobilized (\( d_{x,i} / D_{90} = 1 \)), whereas the finest two fractions have \( d_{x,i} / D_{90} > 1.8 \). For \( d_{x,i} \) larger than 10 or 20 per cent of \( D_{90} \), grains of all sizes are entrained and fractional exchange depths fall within a factor of two, with the very largest fractions tending to have the smallest entrainment.

Comparison of the entrainment with local flow observations (Wilcock et al., 1996) shows that the threshold between negligible grain movement and entrainment of most of the bed surface occurs over a narrow range of local bed shear stress \( \tau_0 \) on the order of 10–15 per cent. A scour depth equal to the thickness of the bed surface
Figure 1. Fractional exchange depth as a function of total exchange depth for large tracer gravel installations on the Trinity River, California: (a) Steelbridge reach; (b) Poker Bar reach. The symbols at each value of $d_x / D_{90}$ represent the mean of three tracer samples. Samples with $d_x / D_{90} < 0.5$ are for a release of $Q = 76 m^3 s^{-1}$; for $d_x / D_{90} > 1.0$, $Q = 164 m^3 s^{-1}$. Mean channel width is 35 m. Fractional exchange depth scales with total exchange depth, although $d_x / D_{90}$ values for the finest fractions tend to be somewhat larger and $d_x / D_{90}$ values for the coarsest fractions are smaller. Under conditions of partial surface mobilization ($d_x / D_{90} < 1$), grains of all sizes are entrained (except the very largest, $D_{98}$) and are in a state of partial transport.

layer occurs at $\tau^* = 0.035$, where $\tau^*$ is the dimensionless shear stress $\tau_{\eta i} (s-1) \rho g D_{\eta i}^{-1}$. $s = \rho_s / \rho$, $\rho_s$ and $\rho$ are sediment and fluid density, $g$ is the acceleration of gravity, and $D_{\eta i}$ is the median grain size of the gravel portion of the bed size distribution. Both entrainment and transport rates decrease rapidly with smaller $\tau^*$ with the bed becoming nearly immobile at $\tau^* = 0.031$. These values of $\tau^*$ are within the range for the critical shear stress of gravel in well-controlled laboratory experiments, demonstrating that local observations of flow and entrainment may be made with similar accuracy in the field (Wilcock et al., 1996).

TRANSPORT COMPONENTS

Calculations of transport rates from tracer observations, as well as comparisons between different tracer measurements, require a formal statement of the relation between transport rate, entrainment and displacement. To facilitate comparison between different sediments and flows, it is useful to express the individual transport components in a form that may be expected to have a time-invariant mean under steady-state transport conditions. An appropriate form is:

$$q_{bi} = M_{\omega i} \left( \frac{N_i}{T} \right) L_{\omega i}$$

where the equation is in units of $[ML^{-1}T^{-1}]$, $M_{\omega i}$ is the mass of fraction $i$ entrained per unit bed area over the time period $T$, $N_i$ is the number of times an individual grain of fraction $i$ is entrained during $T$, and $L_{\omega i}$ is the length of a
single displacement. $M_a$ represents the entrainment of the sediment found in a given bed area at the start of $T$, as would be measured by tracer gravels.

Although each transport component may be represented by a frequency distribution (e.g. Einstein, 1937; Stelczer, 1981; Kirkby, 1991; Hassan et al., 1991), the focus here is their mean values, as defined in Equation 2. The product of means in Equation 2 is equivalent to the ensemble average product for some plausible frequency distributions of the transport components (e.g. an exponential or gamma distribution for $L_{ij}$, as suggested by Einstein (1937), and recently evaluated using tracer gravels by Hassan and Church (1992) and Schmidt and Ergenzinger (1992)). For other possible frequency distributions, the terms in Equation 2 must be regarded as ‘effective’ averages, in the sense that they are defined such that their product gives the mean $q_{bi}$.

Under steady-state transport conditions, $q_{bi}$ varies about a constant mean. $M_a$ may be expected to approach a nearly constant value as the entrainable grains are removed from an area of the bed. Note that this does not mean that entrainment ceases, but that an increasing proportion of the entrainment is composed of grains originating from a different area of the bed. Wilcock and McArdell (1997) found that, under constant flow conditions, $M_a$ approaches a nearly constant value once the cumulative transport exceeds roughly twice the mass of the actively transported bed surface layer. If it is assumed that the mean value of $L_{ij}$ does not vary under steady-state transport conditions, it follows from Equation 2 that $q_{bi}^N/T$ must also have a constant mean value, leading to the plausible assumption that $N_i$ increases directly with time.

In some cases, the basic measured quantity is a combination of the transport components in Equation 2. The rate of entrainment of grains found within a specified unit bed area is given by $M_a/T$ and the instantaneous rate of entrainment per unit bed area (regardless of grain provenance) is given by $M_a N_i/T$. The latter quantity has been measured on films of bedload transport by counting all entrainments from a fixed area over a measured duration (Fernandez Luque and Van Beek, 1976; Drake et al., 1988). The two quantities most commonly observed with tracer gravels are the total displacement length $L_{ti} = N_i L_{ij}$ and the virtual grain velocity $u_{vi} = (N_i L_{ij})/T$. If $q_{bi}$ and $M_a$ are measured, $L_{ti}$ and $u_{vi}$ may be calculated from Equation 2 as:

\[ L_{ti} = \frac{q_{bi} T}{M_a} \]  

and

\[ u_{vi} = \frac{q_{bi}}{M_a} \]  

If $M_a$ is independent of $T$ for steady-state transport, the form of Equation 3 makes it clear that $L_{ti}$ increases directly with the cumulative mass of transported sediment, and therefore with both $T$ and flow strength. In contrast, $u_{vi}$ should take a constant mean for steady-state transport conditions and should depend on flow strength, but not $T$.

Although $M_a$ can be directly measured using the large tracer gravel installations described above, it is useful to define the components of $M_a$ because many tracer observations involve surface grains only and, therefore, may not provide a complete measure of $M_{ai}$. If entrainment occurs only from the bed surface layer, $M_{ai}$ is given by:

\[ M_{ai} = \frac{m_i F_i Y_i}{D_i} \]  

where the equation is in units of [ML$^{-2}$], $m_i$ is the mass of a grain of fraction $i$, $F_i$ is the proportion of fraction $i$ on the bed surface, $D_i$ is fraction size, and $Y_i$ is the proportion of surface grains of fraction $i$ that are entrained over $T$. The number of grains of size $i$, per unit bed area, is approximated by $F_i / D_i^2$, so their mass per unit bed area is $m_i F_i / D_i^2$. Equation 5 is likely to underestimate fractional entrainment at flows larger than those causing full
surface mobilization \((Y_i=1)\) of a fraction. In this case, subsurface grains of that size will also be entrained and \(M_{ai}\) becomes proportional to the exchange depth \(d_x\). For \(Y_i=1\), \(M_{ai}\) becomes:

\[
M_{ai} = \left(\frac{m_i F_i}{D_i^2}\right)^2 d_x
\]

and the term in parentheses represents the mass of fraction \(i\) per unit volume which, when multiplied by the exchange depth \(d_x\), gives the mass of fraction \(i\) entrained per unit bed area.

A smooth transition between the expressions for \(M_{ai}\) at partial transport (Equation 5) and fully mobilized transport (Equation 6) may be obtained if it is assumed that \(d_x \approx D_i\) for some intermediate value of \(Y_i\). A plausible model takes the exchange depth for a given flow to be equal to the grain size of the fraction for which half of the surface grains are entrained \((Y_i=0.5)\) at that flow. In order that \(d_x\) increase consistently with flow strength, it is necessary that the size for which \(Y_i=0.5\) also increase with flow strength, which has been demonstrated for a poorly sorted laboratory sediment (Wilcock and McArdell, 1997). With these assumptions, \(M_{ai}\) is given over the full range of transport as:

\[
M_{ai} = \left(\frac{m_i F_i Y_i}{D_i^2}\right)^2 d_x
\]

with

\[
\Delta_i = \begin{cases} 
1 & Y_i \leq 0.5 \\
\frac{d_x}{D_i} & 0.5 < Y_i \leq 1.0 
\end{cases}
\]

VARIATION OF DISPLACEMENT LENGTH WITH GRAIN SIZE

Basic relations

The relations between the transport components can be used to gain some insight into their functional dependence. We focus here on the variation of \(L_t\) with \(D_i\), for which a summary relation of field data is available for comparison (Church and Hassan, 1992). Comparison of Equations 3 and 4 shows that \(L_t\) and \(u_v\) differ by only a factor of \(T\), so many of the conclusions drawn regarding \(L_t\) should also apply to \(u_v\). Comparison of the two parts of Equation 8 suggests that the variation of \(L_t\) with \(D_i\) should be considered separately for partial transport \((Y_i<1.0)\) and fully mobilized transport \((Y_i=1.0)\). For \(Y_i=1.0\), \(M_{ai}\) is given by Equation 6. Using the spherical approximation \(m_i=\pi\rho_s D_i^3\), Equation 3 becomes:

\[
L_{u_i} = \frac{q_{bi} T}{(\pi/6) \rho_p F_i d_x}
\]

Because \(d_x\) is a constant for a given flow, the only terms in Equation 9 that vary with \(D_i\) are \(q_{bi}\) and \(F_i\). It has been observed that the scaled fractional transport rate \(q_{bi}/F_i\) is independent of grain size for sizes smaller than a threshold (e.g. Wilcock, 1992; Wathen et al., 1995) that increases with flow strength (Wilcock and McArdell, 1993). Size independence in \(q_{bi}/F_i\) occurs when the fractional proportion in transport \(p_i\) becomes equal to its proportion in the bed \(f_i\), the condition of equal mobility (Parker et al., 1982). Because \(q_{bi}=p_i q_b\), where \(q_b\) is the total transport rate, \(q_{bi}/F_i\) takes the constant value of \(q_b\) for all fractions with \(p_i f_i=1\). The relation is approximate because \(p_i f_i\) for equally mobile fractions actually exceeds one as long as \(p_i f_i\) is less than one for any coarser fractions. The onset of equal mobility is closely associated with complete surface mobilization, occurring within one order of magnitude of transport rate of \(Y_i=1\) (Wilcock and McArdell, 1997). For the equally mobile fractions within a mixture, \(q_{bi}/F_i\), and, therefore, \(L_{u_i}\) should be independent of grain size.
For partial transport conditions, \( M_{ai} \) is given by Equations 7 and 8. Considering the case for \( Y_i < 0.5 \), \( M_{ai} \) is given by Equation 5, so that Equation 3 becomes:

\[
L_i = \frac{q_{bi} T}{(\pi/6) \rho_i F_i D_i Y_i} \tag{10}
\]

For fractions in a state of partial transport, \( q_{bi}/F_i \) decreases rapidly with \( D_i \) (Wilcock and McArdell, 1993, 1997), so the effect of both \( q_{bi}/F_i \) and \( D_i \) in Equation 10 is to cause \( L_i \) to decrease rapidly with grain size. Because \( Y_i \) also decreases with \( D_i \), the decrease of \( L_i \) with \( D_i \) is partially balanced by the appearance of \( Y_i \) in the denominator of Equation 10, although we will demonstrate below that the decrease of \( q_{bi} \) with \( D_i \) is more rapid than that of \( M_{ai} \), so that, from Equation 3, \( L_i \) decreases with \( D_i \).

Based only on the elementary relations for \( L_i \) for fully mobilized transport and partial transport and on the fractional transport rates characteristic of these transport regimes, it may be concluded that \( L_i \) should be independent of \( D_i \) for fully mobilized fractions and should vary inversely with \( D_i \) for fractions in a state of partial transport. Before Equations 9 and 10 can be compared with field data, it is necessary to give careful consideration to the manner in which these data are collected and analysed. Two important issues arise. The first concerns the measurement of \( L_i \) using tracers placed initially on the bed surface only, so that no subsurface entrainment is measured. The second concerns the choice of using both mobile and immobile grains or mobile grains only when calculating \( L_i \).

For fully mobile fractions, the displacement length determined using only surface tracers should be larger than that calculated by Equation 9, because the total displacement length of surface grains should be greater than the total displacement length for grains originating from both the surface and subsurface (Hassan and Church, 1992). For fully mobilized fractions, the relation between transport rate and the displacement of surface grains is found by setting \( \Delta_i = 1 \) in Equation 7, so that Equation 3 becomes:

\[
L_i = \frac{(q_{bi})_s T}{(\pi/6) \rho_i F_i D_i} \tag{11}
\]

where \((L_i)_s \) and \((q_{bi})_s \) are the displacement length and transport rate for surface grains only. For fully mobilized fractions, the total transport rate \( q_{bi} \) will include grains that originated from both the surface and subsurface, so that \((q_{bi})_s < q_{bi} \). The relative magnitude of \((L_i)_s \) may be determined by examining the ratio of Equation 11 and Equation 9:

\[
\frac{(L_i)_s}{L_i} = \left( \frac{d_x}{D_i} \right) \left( \frac{(q_{bi})_s}{q_{bi}} \right) \tag{12}
\]

For \((q_{bi})_s < q_{bi} \), Equation 12 may be written as:

\[
(L_i)_s < \left( \frac{d_x}{D_i} \right) L_i \tag{13}
\]

Equation 13 may then be combined with Equation 9 to define bounds for \((L_i)_s \):

\[
\frac{q_{bi} T}{(\pi/6) \rho_i F_i d_x} < (L_i)_s < \frac{q_{bi} T}{(\pi/6) \rho_i F_i D_i} \tag{14}
\]

Although the exact relation between \((L_i)_s \) and \( L_i \) is unspecified, it is seen that \((L_i)_s \) is inversely proportional to \( D_i \) and larger than \( L_i \) by a factor between one and \( d_x/D_i \). For partially mobile fractions, there is likely to be little...
or no subsurface entrainment, so \((L_{ui})_x = L_{ui} \) and \((q_{hi})_x = q_{hi} \) and, to a first approximation, the same result should be obtained regardless of the use of subsurface tracers.

The second necessary consideration when evaluating field measurements of \(L_{ui} \) concerns the inclusion of immobile grains in the calculation of \(L_{ui} \), which is important for all fractions with \(Y_i < 1 \). The value of displacement length calculated using both mobile and immobile grains, \((L_{ui})_{all} \), will be smaller than the value calculated using only mobile grains and is given by:

\[
(L_{ui})_{all} = Y_i L_{ui} \tag{15}
\]

The inferences drawn regarding the variation of \(L_{ui} \) with \(D_i \) may be summarized as follows. For grains in a state of equally mobile transport, \(L_{ui} \) is independent of \(D_i \) and scales with the exchange depth. For grains in a state of partial transport, \(L_{ui} \) decreases rapidly with grain size if \(q_{ui}/F_i \) decreases more rapidly with \(D_i \) than \(M_{ui} \).

For fully mobilized fractions, the observed \(L_{ui} \) will be greater than that given by Equation 9 by a factor of between 1 and \(d_i/D_i \) if \(L_{ui} \) is calculated using only surface tracers. For partially mobile fractions, the observed \(L_{ui} \) will be smaller than that given by Equation 10 by a factor of \(Y_i \) if \(L_{ui} \) is calculated using both mobile and immobile tracers.

**Calculation of \(L_{ui} \) from transport and entrainment observations**

The total displacement length may be calculated from Equation 3 using measurements of fractional transport rate and bed entrainment. We have measured these quantities in a laboratory experiment (Wilcock and McArdell, 1997), thereby providing the opportunity to illustrate the variation of \(L_{ui} \) with \(D_i \) in a fashion that, while indirect, makes use of higher quality observations than are often achievable in the field. The laboratory observations include direct observation of partial transport, so that the effect on \(L_{ui} \) of the degree of mobilization may be demonstrated, supporting the physical interpretation of \(L_{ui} \) developed above.

Recirculating flume experiments were conducted with a widely sorted sediment with \(D_{s0} = 5.3 \) mm, a size distribution extending from 0.21 mm to 64 mm, one-third finer than 2.0 mm, and a median size of the portion greater than 2.0 mm equal to 13.5 mm (Wilcock and McArdell, 1993). The sediment was split into 14 size fractions, each of which was painted a different colour to permit measurement of the surface size distribution from point counts on photographs of the bed. The coloured sediment also facilitated the observation of the proportion of each grain size remaining immobile over the duration of the flume runs (Wilcock and McArdell, 1997). The results of four flume runs are used here. Mean bed shear stress varied between 2.0 Pa and 7.3 Pa; the total transport rate varied between 7.5 g ms\(^{-1}\) and 572 g ms\(^{-1}\). Detail on the sediment, experimental methods, and surface size distribution measurements may be found in Wilcock and McArdell (1993).

The mobile proportion \(Y_i \) of surface grains of all sizes larger than 4.0 mm was measured on time series of bed photographs by noting the presence or absence of individual clasts on the bed surface. An initial set of surface grains of each size was recorded by making drawings of individual grains on projections of photographs taken shortly after the beginning of each run. For fractions in a state of partial transport, the mobile proportion was found to increase rapidly over an initial period and then asymptotically approach a limiting value that varied little with additional run duration. These asymptotic values were used to represent the mobile proportion \(Y_i \) of each fraction. Further detail on the entrainment measurements, the time-dependence of surface mobility, and the variation of \(Y_i \) with flow strength and grain size (which demonstrate the nature and domain of partial transport) may be found in Wilcock and McArdell (1997).

The variation of \(Y_i \) with grain size and flow strength is given in Figure 2a. Partial transport is seen to occur over a range in grain size of approximately a factor of four; the boundaries of the size range increase with flow strength. The corresponding values of \(D_{s0} \) are calculated from Equation 8 and shown in Figure 2b. Exchange depth \(d_i \) is approximated as the size of the fraction observed to be 50 per cent mobile, whose variation with \(\tau_{0} \) can be represented by the power relation (Wilcock and McArdell, 1997):

\[
\frac{d_i}{D_{s0}} = 397(\tau_{50}^{*})^{15} \tag{16}
\]

Figure 2. Entrainment and transport observations as a function of grain size from four flume runs with the BOMC sediment. (a) Proportion $Y_i$ of the surface mobilized over the duration of each run. (b) Value of $\Delta_i$ calculated from Equation 8 using observed $Y_i$ and $d_i$ estimated from Equation 16. (c) Entrainment per unit bed area calculated from Equation 7 using values of $\Delta_i$ in (b). (d) Entrainment per unit bed area scaled with the proportion of each fraction on the bed surface $F_i$. (e) Fractional transport rates. (f) Fractional transport rates scaled with $F_i$.

Partial transport ($Y_i < 1$) occurs over a finite range of sizes of approximately a factor of four; this size range increases with flow strength. Partial transport is associated with rapid decrease with grain size in both entrainment and fractional transport rate

where $\tau_{50}^*$ is the dimensionless shear stress formed using $D_{50}$ of the sediment mix. $M_{ai}$ is calculated using Equation 7 with measured values of $F_i$ and values of $\Delta_i$ from Figure 2b. The measured fractional transport rates are given in Figure 2e. $M_{ai}$ and $q_{bi}$ are scaled by $f_i$ in Figures 2d and 2f. The scaled versions of $M_{ai}$ and $q_{bi}$ are useful because they directly show the variation with $D_i$ of $M_{ai}$ and $q_{bi}$ by eliminating the influence of the proportion of each fraction on the bed surface.

For each run, the largest fully mobile grain ($Y_i = 1$; Figure 2a) is shown by a large grey symbol. The largest equally mobile fraction (operationally defined as $p_i \geq 0.9 F_i$) is marked on Figures 2e and 2f, which clearly show the association of partial transport, fractional equal mobility, and the size-dependent decrease in transport rate. Fully mobile and equally mobile transport occur at similar values of transport rate. Note that eight orders of magnitude are plotted on the transport axis in Figures 2e and 2f. The decrease of $q_{bi}$ with $D_i$ is more rapid than that of $M_{ai}$ (Figures 2c and 2d), showing that $L_i$ will also decrease with $D_i$, according to Equation 3.

$L_i$ is calculated using Equations 7, 8 and 16 in Equation 3, along with measured values of $d_{bi}$, $T$, $\tau_0$ and $Y_i$. As suggested by Equations 9 and 10, $L_i$ is independent of $D_i$ for equally mobile fractions and decreases rapidly with $D_i$ for fractions in a state of partial transport (Figure 3). This is particularly clear in Figure 3b, where $D_i$ is scaled by the size of the largest equally mobile fraction $D_{em}$ and $L_i$ is scaled by $L_{em}$ for that fraction. Recalling that
Figure 3. Total displacement length $L_t$ calculated using Equation 3 with values of $M_i$ from Figure 2c and $q_b$ from Figure 2e. (a) $L_t$ as a function of grain size. (b) $L_t$, scaled by the displacement length of the largest equally mobile fraction, as a function of grain size scaled by the size of the largest equally mobile fraction. The scaled displacement lengths show that $L_t$ is independent of $D_i$ for fully mobile fractions and decreases rapidly with $D_i$ for partially mobile fractions.

$L_t = N_i L_i$, the decrease of $L_t$ with $D_i$ can be attributed to an even more rapid decrease of $N_i$ with $D_i$ because $L_t$ has been suggested to be directly proportional to $D_i$ (Nakagawa et al., 1982; Drake et al., 1988).

Comparison with field data

Church and Hassan (1992) have developed a summary relation for the variation of $L_t$ with $D_i$ using the best available field tracer gravel data. To account for differences in transport rate and flow duration, $L_t$ was scaled using the displacement length of the fraction containing the median size of the bed surface. To account for differences in bed material size distribution, $D_i$ was scaled using the median size of the bed subsurface.

Values of $L_t$ calculated from the laboratory experiment are compared in Figure 4 with the summary relation of Church and Hassan (1992). In Figure 4a, grain size has been scaled using $D_{50}$ of the bulk mix (5-3 mm), which should be similar to the subsurface $D_{50}$ used for the field data. Fractional displacement lengths have been scaled using the displacement length $L_{tg}$ for $D_i=13.5$ mm, which is the median size of the gravel portion of the mix ($D_i > 2$ mm), which was chosen to approximate the field case for which surface $D_{50}$ was 1.5 to 3 times the subsurface $D_{50}$.

For the two largest transport rates, the scaled displacement lengths calculated for the BOMC runs fall within or near the trend of the field data (Figure 4a). The largest fully mobile grain for these two runs falls near the break in slope of the field relation, suggesting that the decrease in scaled displacement length observed for field data with $D_i/D_{50} > 2$ may be associated with partial transport. Scaled $L_t$ for the two runs with smaller transport rates do not follow the field trend, but are displaced towards larger displacement lengths and smaller grain sizes. This is the immediate result of very small calculated displacement lengths for the 13.5 mm fraction for these two runs. Values of $Y_i$ for the 13.5 mm fraction are 0.15 and 0.58 for these two runs, demonstrating that at least one-third of the gravel clasts exposed on the bed surface remained immobile over the duration of the run. This further suggests that the field data may represent cases for which at least the finer half of the bed surface is completely mobilized. If data from smaller transport rates were included in the field compilation, so that partial transport extended to the median and finer fractions, the scaled displacement length for the smaller sizes might...
Figure 4. Scaled total displacement length as a function of scaled grain size. Relation of Church and Hassan (1992) for field observations is shown as the solid line; dashed lines are their estimate of 95 per cent confidence error bars for the relation. (a) $L_s$ scaled by the displacement length of the fraction containing the mean of the gravel portion of the bed; $D_i$ scaled by the median size of the entire sediment mix. (b) $L_s$ scaled by the displacement length of the largest equally mobile fraction; $D_i$ scaled by the size of the largest equally mobile fraction and multiplied by 2.2 to account for the fact that the Church and Hassan relation takes a value of one at a scaled grain size of 2.2.

The grey curve in (b) bounds the data trend that would be obtained if the laboratory displacement lengths were measured using only surface tracers and all tracers, mobile and immobile, were included. The scaled displacement lengths calculated from the laboratory observations fall closely about the field trend. This coincidence, the choice of scaling, the arguments associated with Equations (9) and (10), and the direct observation of partial transport in the laboratory case, all suggest that the field relation represents fully mobilized transport for smaller grain sizes and partial transport for larger grain sizes.

be expected to be larger than the given trend and to decrease more rapidly with grain size. Both of these factors would produce considerably more scatter in the compilation of field displacement data.

The relation between partial transport and the field trend is more evident in Figure 4b, for which the laboratory data are scaled as in Figure 3b. In Figure 4b, the size ratio is multiplied by 2.2 to make the laboratory case consistent with the field relation, for which the scaled displacement length takes a value of one at $D_i/D_{50} = 2.2$. Also shown in Figure 4b is the trend for $L_s$ that would result for the laboratory data if only surface tracers were used and if both immobile and mobile grains were used to calculate $L_{sp}$ as done by Church and Hassan (1992). Although Equations 14 and 15 produce a different relation for each run, the curves are very similar in the scaled domain of Figure 4b, so only a single mean relation is shown. For $D_i < D_{em}$, the true surface-only relation for the laboratory data should fall between the plotted data and the surface-only curve. The laboratory data follow the field trend closely for all sizes and runs. The similarity in shape of the field curve and the laboratory data, together with the direct observation of partial transport and the relative location of the largest fully mobile fraction on each curve, suggest that the field summary of displacement lengths represents fully mobilized transport for finer fractions and partial transport for coarser fractions.

**DISCUSSION**

The spatial variability of bed composition, flow and transport in natural streams requires that comparison between the laboratory data and field relation in Figure 4b be made carefully. The laboratory data are for an essentially uniform transport field. In the field, local entrainment may vary widely and the observed displacement lengths can represent a combination of fully mobilized and partial transport in different locations. Although this could be considered to be partial transport in a spatially averaged sense, important differences can arise relative to the uniform transport case.

Consider the extreme but simple example in which the coarsest fraction $D_{\text{max}}$ in the central one-half of the bed is almost completely mobilized, whereas the remaining half of the bed along the channel margins is completely immobile. In this case, the spatially averaged mobilized proportion is 0.5 for all fractions. According to Equation 5, $M_{ai}$ for each fraction would be given by $[0.5(\pi/6)\rho_s F_i D_i]$. When combined with observed values of $L_{i0}$, this value of $M_{ai}$ would underestimate the transport rate of all fractions except the largest, because the subsurface mobilization of the finer fractions in the central portion of the channel has not been accounted for. Entrainment in the central portion of the channel is actually given by Equation 6 as $[(\pi/6)\rho_s F_i D_i]$ with $d_i$ approximately equal to $D_{\text{max}}$. When reduced by half to account for the immobile portion of the channel, the spatial average of $M_{ai}$ is $[0.5(\pi/6)\rho_s F_i D_{\text{max}}]$, showing that the underestimate of $M_{ai}$ made by using $Y_i=0.5$ is given by $D/d_i$. To accurately calculate $M_{ai}$ and fractional transport rates, it is necessary to know the spatial distribution of $Y_i$ and $d_i$ and calculate the total transport rate as the spatial integral of the local transport rate.

Calculation of transport from entrainment and displacement observations reflects a spatial average over the distance of displacement. In contrast, calculation of displacement lengths from entrainment and transport observations gives a local measure of displacement at the location of the entrainment and transport measurements. Any spatial variation in transport rate will have a corresponding variation in entrainment and displacement length, so that the actual displacement lengths will differ from their local estimate if the transport field is spatially variable.

The influence of grain shape was not systematically measured in the BOMC laboratory work, nor is it accounted for in the summary relation of Church and Hassan (1992). In a study of 480 magnetic-core tracers during two moderate flood events on a step-pool channel, Schmidt and Ergenzinger (1992) observed similar displacement frequencies and distances for tracer grains of rod, ellipsoid and spherical shape, but found that the entrainment frequency of discs was as little as half and the mean displacement length was a factor of three smaller than the other shape classes. The BOMC grains fall within the sphere and ellipsoid classes reported by Schmidt and Ergenzinger, and shape information is not available for all of the data summarized by Church and Hassan. Most of the scatter in their summary plot falls within a factor of three; uncontrolled variation in grain shape is one of several likely causes of this scatter.

CONCLUSIONS

Tracer gravels may be installed in a gravel bed by inserting a large cylinder into the bed and replacing the sediment within with marked grains of the same size distribution. The mass difference of tracers before and after a flow event gives a direct measure of the entrainment per area for each size, which may be directly used in calculations of sediment transport rate. The tracers also provide a local measure of the proportion and size distribution of mobilized sediment, giving information on the size-selective exchange of sediment between flow, bed surface and subsurface, which is needed to understand bed armouring, selective deposition and subsurface flushing. Installation of tracers below the depth of scour eliminates uncertainties associated with estimating entrainment from surface tracers alone. The labour involved in replacing a section of bed with tracer gravels is considerable, although partly offset by the fact that size distributions of the surface and subsurface are obtained in the process. Large tracer gravel installations may be used in combination with scour chains or other scour depth indicators to provide more measurements and broader coverage along with an estimate of the mass and size distribution of entrained sediment.

Calculation of transport rate from tracer gravel observations requires an explicit statement of the relation between transport rate, entrainment and displacement length. The relation also provides a basis for evaluating tracer data collected by different methods. When calculating transport rates, particular attention must be given to whether the tracers are placed only on the bed surface, or in both the surface and subsurface, and to whether both mobile and immobile tracers are included in the calculations. Total displacement length depends on the cumulative product of transport rate and flow duration, so that both flow strength and sediment properties are important, as indicated by any transport rate relation.

The formal relation between transport, entrainment and displacement may be used to provide physical insight regarding the expected behaviour of tracer grains. The variation of displacement length with grain size depends on the degree of mobilization of the individual fractions in the bed. For finer fractions in a state of
equally mobile transport (in which the proportion in transport equals or exceeds that in the bed), total displacement length may be expected to be independent of grain size. For coarser fractions in a state of partial transport (in which a portion of the surface grains remains immobile throughout the flow event), total displacement length should decrease rapidly with grain size. These relations are supported by displacement lengths calculated from observed entrainment and transport rate in a laboratory flume. Comparison of these results with a summary relation for field displacement data suggests that the field relation represents flow and transport conditions for which the finer fractions are fully mobilized, whereas the coarser fractions are in a state of partial transport.

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