Lecture 1 - Sediment Transport -

Basic concepts, sediment rating curves, introduction to transport models

1. Why estimate sediment transport?

(1) Sediment Yield: e.g. impacts on receiving waters (reservoir filling; water quality; mining)

(2) Channel change: bed aggradation/degradation, bank erosion (navigation, flood levels, bridge and bank protection))

In general, channel change is a function of water and sediment supply & the basic sediment transport questions are:

⇒ *How much sediment can the channel transport with the available water?*

⇒ *Is this transport rate greater or smaller than the rate at which sediment is being supplied to a reach?*

Some examples of channel change problems:

(a) Channel design: will the design channel be able to handle the imposed load with the available flow?

(b) Impacts of channel alteration (esp. dams & diversions): what will be the effect & how severe? How far d/s will it extend? Can it be reversed or managed?

(c) Sediment mining: can sediment be removed from the stream w/o causing problems d/s?

(d) Land-use changes (farming, forestry, fire, urbanization): what will be impacts on channel; how quickly will they occur; for how long?

(e) Tributary flooding, landslides

(f) Stream ecology: habitat change is produced via sediment transport; ecological concerns also focus on the frequency of transport (disturbance).

2. Introductory Material

Before getting started, lets introduce some basic terminology and concepts.

Sediment Grain Size

The primary basis for classifying sediment is grain size, for which we use a scale based on powers of two. Although originally defined using the ϕ (phi) scale, where D in mm is

$$D = 2^{-\phi}$$

those of us who deal with coarse grain sizes found the negative sign to be annoying and use, instead, $\Psi = -\phi$. The following table lays out the basic grain size classes.

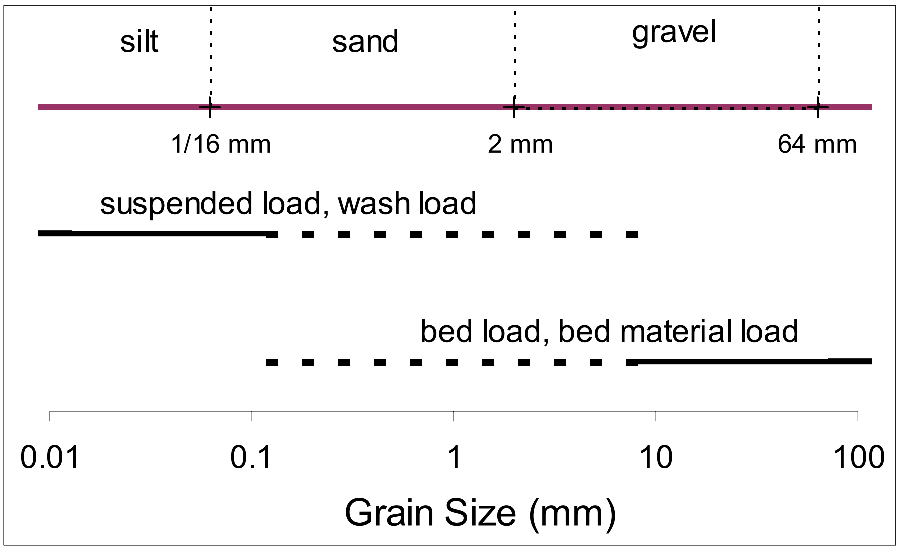
Power of 2 Ψ	2^Ψ from (mm)	to (mm)	Size Class	
	-	0.002	clay	
-9	0.002	0.004	vf silt	
-8	0.004	0.008	f silt	
-7	0.008	0.016	m silt	
-6	0.016	0.031	c silt	
-5	0.031	0.063	vc silt	
-4	0.063	0.125	vf sand	
-3	0.125	0.25	f sand	
-2	0.25	0.5	m sand	
-1	0.5	1	c sand	
0	1	2	vc sand	
1	2	4	vf gravel	pea gravel
2	4	8	f gravel	pea gravel
3	8	16	m gravel	
4	16	32	c gravel	
5	32	64	vc gravel	
6	64	128	cobble	
7	128	256	cobble	
8	256	+	boulder	

Even a cursory examination of real streams demonstrates that the range of sizes in the bed is typically very large. These sizes are not found in a homogeneous mixture, but are sorted into regions of finer and coarser size. In coarse-bedded streams, there is also vertical sorting, wherein the surface of the streambed is generally coarser than the underlying material, which we refer to as armoring. Finally, the distribution of sediment size in a stream bed is not static, but changes with changes in flow and in sediment supply. Because transport rate depends strongly on grain size, we will have to find a way to specify bed-material grain size that effectively represents this spatial and temporal variability.

Some general transport concepts

Sediment transport is often separated into two classes, based on the mechanism by which grains move. These are *bed load*, wherein grains move along or near the bed by sliding, rolling, or hopping, and *suspended load*, wherein grains are picked up off the bed and move through the water column in generally wavy paths defined by turbulent eddies in the flow. In many streams, grains smaller than about 1/8 mm tend to always travel in suspension, grains coarser than about 8 mm tend to always travel as bed load, and grains in between these sizes travel as either bed load or suspended load, depending on the strength of the flow. It is useful to divide transport into these categories because the distinction helps to develop an understanding of how transport works and what controls it.

Sediment transport in streams can also be divided into two other classes, based on the source of the grains. These are *bed material load*, which is composed of grains found in the stream bed, and *wash load*, which is composed of grains found in only small (less than a percent or two) amounts in the bed. The sources of wash load grains are either the channel banks or the hillslope area contributing runoff to the stream. Wash load grains tend to be very small (clays and silts and sometimes fine sands) and, hence have a very small settling velocity. Once introduced into the channel, wash-load grains are kept in suspension by the flow turbulence and essentially pass straight through the stream with negligible deposition or interaction with the bed.



The boundary between bed load and suspended load is not sharp and depends on the flow strength. Consider a stream with a mixed bed material of sand and gravel. At moderate flows, the sand in the bed may travel as bed load; as flow increases, the sand may begin moving partly or entirely in suspension. Even when traveling in suspension, much of this sediment (particularly the coarse sand) may travel very close to the bed, down among the coarser gravel grains in the bed. That makes it very difficult to sample the suspended load in these streams or, for that matter, to even distinguish between bed load and suspended load. This difficulty is one reason why we focus in these lectures on *bed material load*, rather than bed load and suspended load. Another reason is one of simplicity: the bed material in a stream can be defined and measured. We are interested in its transport rate and should invoke the alternative classification—based on transport mechanisms—only if it helps us reach our goal of predicting transport rates.

The concept of bed material load has a couple more features worth noting at this point. One is that we expect that bed material load should be *predictable* in terms of the channel hydraulics and the composition of the bed. Wash load, on the other hand, is generally not predictable based on channel hydraulics and bed composition. To predict the wash load we need to predict the rate at which these fine sediments are supplied to the stream. The other important concept regarding bed material load has to do with the effect of sediment supply on transport rates. If the supply of fine sediment in wash load range is increased, we should observe an increase in the wash load, but the transport rates of the other grain sizes—comprising the bed material—should remain unchanged (unless we add so much wash load material that the flow turns into a thick slurry like pea soup). In contrast, if the supply of bed material is changed, we expect that the bed composition will change and, therefore, that the transport rates of the bed material will also change. For example, if the supply of coarse sand to a gravel-bed stream were increased (this is a common problem), we would expect the amount of these sizes in the bed to increase. We might also expect that the transport rate of these sizes would also increase (if for no other reason than because there is more of sand in the bed) and that the transport rate of other sizes in the bed (e.g. coarse gravel, cobble) to decrease (because their proportion in the bed material is decreasing). It turns out that the actual affect of adding sand to a gravel-bed channel is more complicated than that (we will revisit the subject later), but the basic principle remains valid: altering the supply of sediment in the size range of the bed material will alter the bed composition and the transport rates, whereas altering the supply of sediment in the size range of wash load will have negligible effect on the bed composition and bed material load. These distinctions may seem picky at this point, but they end up being important when trying to understand channel change in response to changes in sediment supply to a stream channel.

The washload/bed-material load approach suggests a breakdown of general size categories for gravel-bed rivers with four parts (see first figure at the end of notes for Lecture I). The finest fraction is wash load, the next three fractions are bed-material load and subdivided according to their transport mechanisms. In gravel-bed rivers, wash load generally consists of clay, silt, and, in many cases fine sand. Fine bed material load typically consists of medium to coarse sand and, in many cases, pea gravel, which can move as either bed load or suspended load. When in suspension, the grain trajectory is typically within a near-bed region where the flow is locally disturbed by wakes shed from the larger grains in the bed. Fine bed material exists in the interstices of the bed and in stripes and low dunes at larger concentrations. The near-bed suspension of the fine bed material cannot be sampled with conventional suspended sediment samplers and models for predicting its rate of transport are incomplete. Coarse bed material forms the framework of the river bed. Its motion is almost exclusively as bed load. Displacements of individual grains are typically rare and difficult to sample with conventional methods. Regardless of transport mechanism, fine and coarse bed material are true bed-material load because the source is the streambed and changes in the proportion of one will affect the transport rate of the other. In some streams, there is another, yet coarser fraction, typically in the boulder size class, that is immobile at typical high flows. These grains can exert considerable influence on the flow and must be included in any form drag evaluation.

In these lectures, we will focus on the transport of bed material in coarse-bedded streams. By coarse-bedded, we mean streams whose bed material includes substantial amounts of gravel or coarser sediment. The bed material may contain abundant amounts of finer material (basically sand) or none at all. Bed material transport is the basic engine of fluvial geomorphology. The balance between sediment supply and sediment transport rates in stream channels governs bed scour and aggradation, channel topography and flow patterns, and the subsequent erosion and construction of bars, bends, banks, and floodplains.

Two constraints

There are two overarching constraints that bound any approach to estimating transport rates in gravel-bed rivers. These are the spatial and temporal variability of the transport process itself, and the sparse information that is typically available for developing an estimate of bed-material transport.

A picnic on the bank of a typical gravel-bed river is sufficient to demonstrate the considerable spatial variability in channel topography, bed material, sediment supply and flow (see four stream pictures at the end of notes for Lecture I). These quantities also vary in time, typically over very different scales. Although this is also a concern for those concerned with the transport of fine sediment in suspension, this transport tends to be much better mixed, indicating that fewer samples may be needed and that a spatial average may be a tractable modeling approach. The transport of bed material in gravel-bed rivers is a decidedly local affair, driven by strongly nonlinear relations controlled by local values of flow velocity and bed material grain size. This variability requires a dense array of large samples for adequate accuracy and makes model predictions based on spatial averages difficult.

The second constraint—sparse information—is, of course, directly related to the first. If there were little spatial variability in the transport, only a few observations would provide the necessary information. The sparse information particularly affects our ability to model. Models that depend strongly on the local details of flow and bed material (e.g. mixed-size transport models using many size fractions) require abundant local information for accurate predictions. This information is seldom available. Although essential for developing our understanding of the relevant processes and useful for exploring general scenarios, such models are fragile: unable to produce reliable predictions from sparse, uncertain input. For practical transport estimates, our models must be *robust*: they must be able to give reliable answers that are not too sensitive to the uncertainty in the available information.

A final point worth noting is that most of the time, very little is happening on the bed of a gravel river and, as we will discuss, even during floods, most of the coarse bed-material grains move only occasionally, in a stochastic.

Objectives

In these lectures on sediment transport, the primary objective is to provide a basis for estimating the rate of transport in coarse-bedded rivers. Although a methodology will be developed, our goal is not to present a simple recipe but to develop an understanding of what how transport works, what controls it, and how these factors can be expressed in a general model or method that can be widely applied with some confidence. We will consider elements of basic transport models, the options for describing flow in these models, and the relative merits of using field measurements or formulas in making our estimates. In the end, I will suggest that there is a need to more fully mesh monitoring and modeling. We cannot measure everything, everywhere. Models are needed to concisely represent our understanding and place it in a testable form. Any prediction of future conditions can only be made with a model. The methods to be used are not settled but the subject of ongoing research. We are headed in the direction of developing reliable models and incorporating these models into a useful, efficient predictive method of demonstrated accuracy. The future of sediment transport estimation will take the form of relatively simple and robust models supported by monitoring methods that provide a high-resolution signal at modest cost.

So, the lectures primarily focus on the first of the two points highlighted in the box on the first page of these notes: *How much sediment can the channel transport with the available water?* In the last lecture, we will make some headway on the second question: *Is this transport rate greater or smaller than the rate at which sediment is being supplied to a reach?* A complete answer to this problem requires a definition of the sediment supply to a reach, information is that is rarely available whose prediction is beyond the scope of our efforts here. What we can cover, however, are the fundamentals of how to balance the sediment transport rate with its supply and what this indicates for the storage or evacuation of sediment from a reach and the accompanying changes in bed composition and channel geometry.

3. Sediment Rating Curves

Most of the sediment transport problems are most obviously defined in terms of the supply of sediment (which includes at least its volume or rate and its grain size) and the water supply, or discharge. Eventually, we want to estimate sediment transport rate Q_s as a function of water discharge Q. A relation giving Q_s as a function of Q is called a **sediment rating curve**. A sediment rating curve typically takes the form of a power function:

$$Q_s = aQ^b \tag{1}$$

where, in the US, Q_s is in units of tons per day and Q is in units of ft^3/s, or cfs. Preferable units would be kg/hr or tonnes per day and m^3/s.

To illustrate: if you have a sediment rating curve such as (1) and a record of discharge (e.g. the daily mean value of Q for 25 years), you can calculate the total sediment load (the sediment yield) by using (1) to calculate the tons of sediment transported for each day and adding up all 9131 or so values to get a total sediment yield for 25 years. {Beware: even if a sediment rating curve can be defined from some transport observations, you can't assume that such a curve will remain the same over time: changes in sediment supply, or channel configuration, or channel bed material, can change the rating curve. Also, discharge may change rapidly on many streams (e.g. small snowmelt-dominated streams; flashy arid and urban streams}, such that daily mean discharge will not accurately represent the flow producing transport. In such cases, a much finer time resolution (e.g. 15 minutes) may be needed.}

If we directly and simultaneously measure both Q_s and Q, we can, with enough observations, develop an empirical sediment rating curve, which usually amounts to determining the slope b and intercept log(a) when fitting a straight line to

$$\log(Q_s) = \log(a) + b\log(Q) \tag{2}$$

Exponentiating (2) gives (1).

But, what if we don't make direct measurements—which are time consuming, risky at high flow, and, by their empirical nature, provide no ability to predict transport under conditions other than those measured? Then, we have to use some kind of model to predict transport rate. This model will hopefully be both practical and general. Practical, meaning that the measurements (*initial* and *boundary conditions*) are obtainable without too much effort. General, meaning that the model, or method, can be counted on to work under most any reasonable set of conditions.

4. Sediment Transport Models

Transport Model Based on Q: Part I

The most important aspect of developing a transport model is developing a basis for scaling, or representing, the discharge Q. Since we often wish to develop a sediment rating curve, the obvious thing to try is to develop a model based directly on Q. Unfortunately, it takes just a little thought to conclude that such a model could not possibly be general. It hopefully seems quite unlikely that, say, 100 cfs would produce the same transport rate in a small creek one could jump across and in a very large river that might be a km wide or more. Differences in channel size, shape, slope, roughness, and bed material composition would produce very different Q_s for the same Q. What this means is that very different values of the coefficient a and the exponent b in (1) would be needed to estimate Q_s in different rivers (and that assumes that the transport is actually well described by a power function, which may not always be the case).

It's too bad that a sediment transport model such as (1) is not available, since it is the basis we need for most problems. The interest in developing a model sediment rating curve between Q_s and Q is sufficiently strong that folks don't really give up. One recent attempt defined a dimensionless version of (1)

$$\frac{Q_s}{Q_{sr}} = \left(\frac{Q}{Q_r}\right)^b \tag{3}$$

This is formed by writing (1) a second time for some reference condition, and then dividing (1) by this second version of the equation, which has the desirable result of causing the coefficient a to cancel. The idea is that one might make a few measurements of transport rate at the reference flow Q_r. With known values of Q_{sr} and Q_r, one could then predict Q_s for any other Q. Unfortunately, even though the coefficient a cancelled in developing (3), the exponent b is still lurking around and a general model would require either a constant value of b (which was proposed) or some independent way of calculating it (for which no method has been suggested). We will later demonstrate that b is not, in fact, a constant, once we have looked into transport models in a little more detail.

Transport Model Based on Bed Shear Stress

A measure of flow strength that has been found to provide a general description of transport rate is the bed shear stress τ. A stress is a force per area: in this case, the shear force exerted by the flowing water on an area of the bed. That the transport should depend on the fluid force applied to the bed should, hopefully, seem reasonable. The price we pay for using τ is that we will have to figure out how to estimate it. That is the subject of the next lecture.

Transport is conveniently treated as a flux per unit width. We define transport rate per unit width q_s as the volume of sediment $\forall_s$ transported per unit time and width [L^2T^{-1}]. To get a feel for the constituents of a general transport model, it is useful to do a dimensional analysis. We can imagine that q_s will depend on a number of variables representing the strength of the flow, the fluid, and the sediment. We use τ to represent the flow strength and we might also include flow depth h in the list (arguing that the relation between q_s and τ might be different for different values of h). We represent the sediment using grain size D and sediment density ρ_s. Both of these control how heavy a grain is, which should influence transport rate, and D also controls the grain area exposed to the flow, which should also influence the transport rate. For now, we will pretend that the sediment contains only one size and leave for the third lecture the difficult problem of representing grain size when you have a mixture of a wide range of sizes. We represent the fluid using water density ρ and water viscosity μ. Density ρ is the fluid mass per volume and governs the interaction between forces and accelerations in the fluid (e.g. for the same τ and D, you can imagine that transport rates in air, which has very low density, would be different than transport rates in water). Viscosity μ describes the resistance of a fluid to deformation (e.g. for the same τ and D, you can imagine that transport rates in a viscous motor oil would be different than transport rates in water). Finally, we need to include the acceleration of gravity g, which influences the movement of both the water and the sediment grains. Our list of variables is then:

$$q_s = f(\tau, h, D, \rho_s, \rho, \mu, g) \tag{4}$$

We will do the dimensional analysis in lecture, which leads to the end result

$$q^* = f(\tau^*, S^*, s, D/h) \tag{5}$$

where

$$q^* = \frac{q_s}{\sqrt{(s-1)gD^3}}, \quad \tau^* = \frac{\tau}{(s-1)\rho g D}$$

$$S^* = \frac{\sqrt{(s-1)gD^3}}{\mu/\rho} \text{ and } s = \frac{\rho_s}{\rho}$$

In short, if we follow the rules of dimensional analysis, we know that the relation among the five variables in (5) contains all the information in the relation among the eight variables in (4). Now, if we are only concerned with quartz density grains in water (we are excluding transport in air!), and flow depths greater than a few times the grain size D, and grains coarser than a half mm or so, then we can neglect the last three variables in (5), leaving only two variables behind, q^* and τ^*. Each has a nice physical interpretation. The transport variable q^* can be shown to represent the ratio of the volumetric transport rate q_s to the product (wD), where w is the grain fall velocity. q^* is commonly called the Einstein transport parameter. The dimensionless shear stress τ^* represents a ratio of the shear stress (flow force per area) acting on the bed to the grain weight per area. τ^* is widely known as the Shields Number.

Dropping S^*, D/h and s from the list in (5), we are left with

$$q^* = f(\tau^*) \tag{6}$$

which says, in essence, that the rate of transport (relative to grain size and fall velocity) will depend on the flow shear force (relative to the grain weight). Hopefully that makes sense. Transport functions often take a power form like

$$q^* = a(\tau^* - \tau_c^*)^b \tag{7}$$

where

$$\tau_c^* = \frac{\tau_c}{(s-1)\rho g D} \tag{8}$$

and τ_c is the critical value of τ necessary for getting the grain moving. The quantity (τ^* - τ^*_c) is an expression for the "excess" shear (another is $\{\tau^*/\tau^*_c\}$) above critical. For example, a well known empirical bed-load function is the Meyer-Peter and Müller (M-PM) formula:

$$q^* = 8(\tau^* - \tau_c^*)^{3/2} \tag{9}$$

In the third lecture, we will explain that the critical shear stress τ_c is difficult to both define and measure for unisize sediment and nearly impossible to measure for mixed-size sediments. For most purposes, it is both reasonable and useful to define a surrogate for τ_c, the reference shear stress τ_r, which is the shear stress that produces a small, constant, and agreed-upon reference transport rate. By its definition, τ_r should be close to, but slightly larger than τ_c. Before doing that, it is useful to define a new dimensionless transport parameter

$$W^* = \frac{q^*}{(\tau^*)^{3/2}} = \frac{(s-1)gq_s}{(\tau/\rho)^{3/2}} \tag{10}$$

It is worth noting that W^* does not contain the grain size D, which we will see later is an essential feature when developing a general model for the transport rates of sediments of different size or for different size fractions within the same mixture. The reference transport used is $W^* = 0.002$. To see how this works, lets recast the M-PM formula using a reference transport rate. First, we divide (9) by $(\tau^*)^{3/2}$ to get

$$W^* = 8\left(1 - \frac{\tau_c^*}{\tau^*}\right)^{3/2} \tag{11}$$

Now, solve (11) for the value of τ^* at $W^* = W^*_r = 0.002$. Dividing by 8, raising both sides to the 2/3 power produces

$$0.004 = 1 - \frac{\tau_c^*}{\tau_r^*} \tag{12}$$

from which we see that $\tau_c^* = 0.996\tau_r^*$ (13)

and that τ^*_r is slightly larger than τ^*_c, as desired. Using (13) to replace τ^*_c in (11), we get

$$W^* = 8\left(1 - 0.996\frac{\tau_r^*}{\tau^*}\right)^{3/2} \tag{14}$$

which gives the M-PM formula in terms of the reference shear stress.

5. Review & Application

Lets review what we have covered, using the Meyer-Peter and Müller (M-PM) function for illustration. We would like to find a general transport function and, unfortunately, the flow has to be described by something other than the flow discharge Q, which would be most convenient. It turns out that the boundary shear stress τ works well. To be general, the transport function should be independent of the system of units and should include all relevant variables. Dimensional analysis gets us to the point where we think that two dimensionless variables q^* and τ^* might capture the essence of the problem, at least for flows that aren't too shallow and for grains that aren't too small. An example of such a relation is the M-PM formula

$$q^* = 8(\tau^* - \tau_c^*)^{3/2} \tag{9}$$

or, written out,

$$\frac{q_s}{\sqrt{(s-1)gD^3}} = 8\left(\frac{\tau}{(s-1)\rho g D} - \frac{\tau_c}{(s-1)\rho g D}\right)^{3/2} \tag{18}$$

If we solve (18) for q_s we get

$$q_s = \frac{8}{(s-1)g\rho^{3/2}}(\tau - \tau_c)^{3/2} \tag{19}$$

Before going further, we immediately note an unusual thing: despite claims that D is important in determining transport rate, we find that it is not even present in (19)! So, how can D influence the transport rate? The answer is that D controls the critical stress τ_c: bigger grains tend to have a larger τ_c. We will discuss estimating τ_c in the third lecture. If we can estimate τ_c, and have some way of estimating τ (from Q, for example), we can use (19) to calculate q_s. We then multiply q_s by the channel width b (or more accurately, the width of the channel containing bed material) to determine the total transport rate through the section. Not as direct as having a formula based directly on Q_s and Q, but at least the method appears to be general.

Example. [see "SimpleMPM.xls] Use the M-PM formula to estimate transport rate in a 20 m wide stream. The shear stress τ is 19.6 Pa (which corresponds to a simple flow with depth of 1 m and slope of 0.002) and the critical shear stress τ_c is 16 Pa (which corresponds approximately to 22mm gravel).

Answer. We need some additional values to complete the calculation. The relative density s of quartz is 2.65 and s for most minerals is generally within 5% of this value. The density of water ρ is 1000 kg/m^3 and varies little. Gravity g is 9.81 m/s^2, which is all you need unless calculating transport on Mars or Venus. Solving (18) for q_s and inserting the given values of all parameters gives q_s = 0.000107 m^2/s. Multiplying by the width b gives Q_s = 0.00213 m^3/s. This might seem like a tiny number, but then a cubic meter of sediment per second is a lot of sediment. It might be more useful to express Q_s in units of kg/hr. The conversion is

$$0.00213\frac{m^3}{s}\left(\frac{2650kg}{m^3}\right)\left(\frac{3600s}{hr}\right)=20{,}330\frac{kg}{hr}$$

which is actually a pretty large transport rate. For reference, values of various dimensionless variables for the problem are

$$\tau^*=0.055 \quad \tau_c^*=0.045 \quad q^*=0.00812 \quad W^*=0.628$$

6. Concluding Thoughts

Finally, we consider the form of the MP-M function, because it reveals why transport rates are so extremely difficult to estimate. Although the power function approaches an exponent of 3/2 at large τ^*, it also becomes arbitrarily steep as τ^* approaches τ^*_c. Unfortunately, most transport in coarse-bedded rivers occurs at τ^* not much larger than τ^*_c (even during floods, τ^*/τ^*_c often does not exceed two), so we are dealing with a very steep nonlinear function, as shown on the following plots of the M-PM function. What this means is that small errors in τ^* (or τ^*_c) can produce enormous errors in estimated transport rate. Recalling that τ^* contains τ in the numerator and D in the denominator, we can summarize our difficulties in terms of uncertainty in estimating τ (the flow problem, the second lecture) and uncertainty in estimating D (the sediment problem, the third lecture).