Lecture Notes - Sediment Transport – Incorporating Water & Sediment Supply

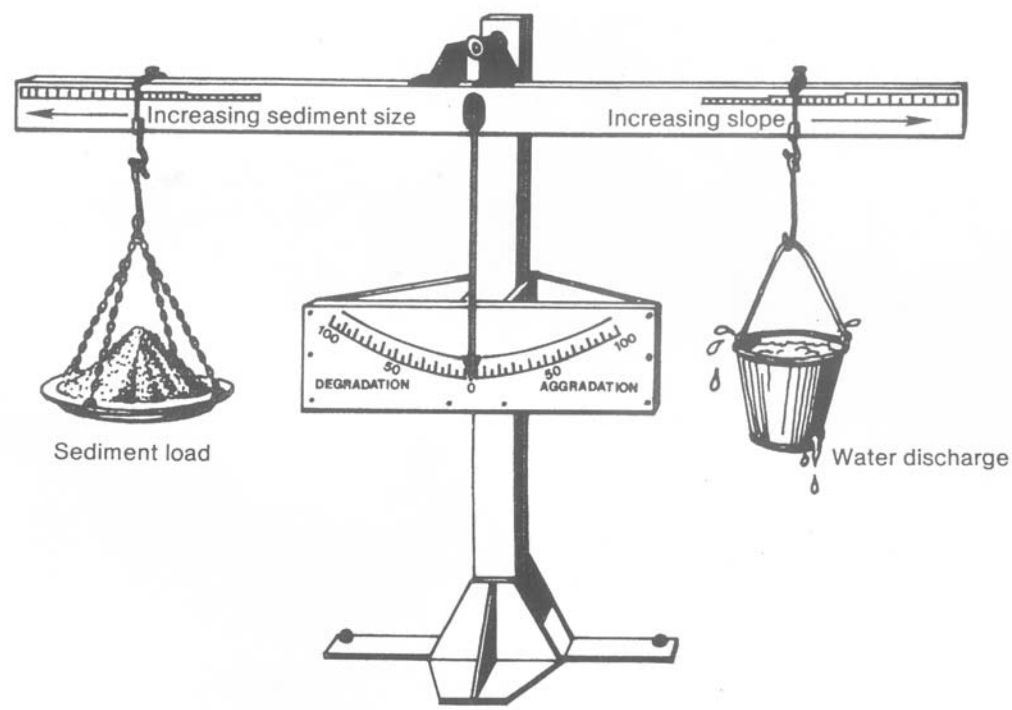
At the beginning of the first lecture, we introduced two basic sediment transport questions:

⇒ *How much sediment can the channel transport with the available water?*

⇒ *Is this transport rate greater or smaller than the rate at which sediment is being supplied to a reach?*

We have spent all of our time up until now dealing with the first question. This lecture provides a brief introduction to the second. The essential idea here was very nicely presented a half century ago in the "stable channel balance" by Lane (Lane, E.W., 1955. Design of Stable Channels. *Transactions, Am. Soc. Civil Eng.* 120:1234.) We will discuss this diagram in class and much of the lecture is intended to provide a quantitative interpretation for it.

The most important point, which I will write down in order to make sure I do not forget mentioning it, is that the diagram is most clearly interpreted by considering the water discharge and the sediment load as that *supplied to* a channel reach. That is, the sediment load in the balance is not the *transport capacity* that we might calculate for the reach given the discharge and the channel size, slope, and bed material. In fact, it is the difference between these two, the sediment supply and the transport capacity, that determines whether the reach will aggrade or degrade.



The Controls of Channel Geometry

Much has been written in the geomorphology and river engineering literature about the controls of channel geometry. Essential to the debate are the questions "what is an independent variable?" and "what is a dependent variable?". It turns out that the choice depends on the scale of space and time one is worried about. At large time and space scales (river basins over millennia, for example), some variables (like valley slope) become dependent variables whereas they must be accepted as independent variables, or imposed on the problem, at time scales approaching several years to several decades. We will not fully engage in this debate here, but will consider a case spanning a river reach and a time period of perhaps a decade or two (e.g. a period encompassing a half dozen or so significant flows). We use this time scale because it is at the heart of the channel change problem (whether induced by natural or human causes) and because the interactions among the variables involved are clear enough and allow us to invoke the underlying physical relations with at least approximate accuracy.

If one were designing a river channel that you hope will hold together for a couple decades (at least until the end of your professional career), it seems clear that the absolute minimum set of properties you would need to determine would be the size of the channel (its width b and depth h) and its slope S. You would also need to specify the sediment composing the channel, which we will represent with only grain size D for this simple analysis. Finally, you would have to specify some kind of design discharge Q or, more usefully, a time series of discharges, that the channel would carry, along with the sediment supply Q_s that the discharge would deliver to your channel. So we set the problem up as

$$(b,h,S) = f(Q,Q_S,D) \tag{1}$$

For this simple analysis, we will not initially worry about any difference in grain size between the bed material and the sediment supply, nor will we worry about other variables, such as valley slope and channel cross-sectional shape and planform.

Eqn. (1) indicates that we will need 3 equations in order to predict the three unknowns b, h, and S. In open channel flow, the two relations that are always available are standard equations for the conservation of fluid mass and momentum. For steady, uniform flow, these are

$$Q = UA \tag{2}$$

$$\tau_0 = \rho g R S \tag{3}$$

Although we have two equations in our toolbox, we immediately notice that we have also introduced two new variables, the mean velocity U and the boundary shear stress τ_0. Because neither U nor τ_0 can be specified in advance, we are no closer to our goal. We now have two equations, but we now need *five* equations to predict the five unknowns b, h, S, U, and τ_0. We do have two other relations to call on, one for flow resistance (we will use Manning's equation) and one for sediment transport rate (we will use the Meyer-Peter & Müller relation).

$$U = \frac{\sqrt{S}}{n} R^{2/3} \tag{4}$$

$$\frac{q_s}{\sqrt{(s-1)gD^3}} = 8\left(\frac{\tau}{(s-1)\rho g D} - \frac{\tau_c}{(s-1)\rho g D}\right)^{3/2} \tag{5}$$

Now, we have four relations to predict our five unknowns. The last, missing relation has been the subject of longstanding research in fluvial geomorphology and river engineering. This has often been called the "width problem", in the sense that the available relations (eqns. 2 through 5) work well for determining channel depth and slope in a channel of specified width or on a per-unit-width basis. In essence, our last, missing relation would provide some indication of how big the channel would be. Many have claimed success in developing such a relation, but no approach is uniformly accepted. The difficulty in predicting channel width is, in my opinion, one of the two most important unsolved problems in fluvial geomorphology (the other is predicting sediment yield).

For this introduction, we will simply specify channel width (thereby removing it from the list of variables that must be predicted) and consider how our four governing equations indicate how flow depth h and, most importantly, channel slope S, vary with b, along with Q, Q_s, and D. To do this, we will use the spreadsheet "ChanSim.xls", the workings of which we will discuss in class.

To interpret the results of these calculations, we have to understand that the solution to these equations gives the *equilibrium* h and S. That is, for a specified combination of b, Q, Q_s, and D, the simultaneous solution to equations (2) through (5) gives us values of h and S that would keep the channel in equilibrium. The important distinction concerns the sediment supply Q_s. If this Q_s is used in the M-PM equation, then we are calculating the combination of h and S that will produce a *transport capacity* in the channel (for the given b, Q, and D) that is equal to the the sediment supply. Thus, the channel has no aggradation or degradation and is at equilibrium. Also, if the initial transport capacity does not equal the sediment supply, our development here does not tell us anything about the details of channel adjustment and how long the adjustment will take, although it does provide a quantitative basis for considering possible channel changes.

So, how do we link these calculations back to the Lane sediment balance? The key element in interpreting the results of these calculations concerns the channel slope S. We will work on this concept with the flume, but you can also visualize the process using a simple thought experiment. Consider a simple, rectangular channel with a sediment bed. A water and sediment supply Q and Q_s are imposed on the channel and the bed is at equilibrium, neither aggrading nor degrading over time. Now, what would happen if we increased Q_s? The sediment would be coming in at the upstream end of the flume faster than it could be transported with the available Q. Sediment would start accumulating at the upstream end of the channel. This means S would gradually increase, as the sediment bed accumulated toward the upstream end. The increase in S also indicates how the channel would find a new equilibrium. As S increases, so does τ_0 through (3). So, bed aggradation would occur until the new S produced a τ_0 that was just sufficient to transport the new, larger supply Q_s.

In the case of a decrease of sediment supply, just the opposite would happen. For the initial S, the transport capacity at the upstream end of the channel would be greater than the new, smaller sediment supply. Sediment would be entrained from the bed faster than it was being replenished from the sediment supply. The bed slope S would decrease and would continue decreasing until an equilibrium S produced a τ_0 that was just right to match the new, smaller supply Q_s.

When interpreting the calculations in ChanSim.xls, Q_s is understood to be the sediment supply to the reach and an increase in S represents bed aggradation and a decrease in S represents degradation, thus linking back to Lane's balance. Thus, we have a tool that indicates the tendency of a channel to aggrade or degrade, given forecast changes in Q and Q_s.

We can also develop a simpler, one-function version of the same kind of analysis, developed by Henderson (*Open Channel* Flow, Macmillan, 1966). We start with a useful, but approximate transport formula

$$q^* c \tau^{*3} \tag{6}$$

where c is a constant. This is known as the Einstein-Brown formula (basically, it is an approximation, introduced by Brown, of the Einstein bedload function, appropriate for common sediment transporting flows in many channels). To be clear, lets write out the formula in dimensional terms

$$\frac{q_s}{\sqrt{(s-1)gD}} = c\left(\frac{\tau}{(s-1)\rho g D}\right)^3 \tag{7}$$

We approximate τ using the depth-slope product ρgRS. Putting this in (7) and using a proportionality to neglect the constant terms

$$q_s \propto \frac{(RS)^3}{D^{3/2}} \tag{8}$$

Now, we introduce continuity and flow resistance to replace R with q, the discharge per unit channel width. For simplicity, we will use the Chezy flow resistance equation

$$U = C\sqrt{RS} \tag{9}$$

and write continuity as

$$q = Uh \approx UR \quad (10)$$

Combining (9) and (10) we get

$$q \propto R^{3/2}\sqrt{S} \quad (11)$$

So,

$$R^3 \propto q^2 / S \quad (12)$$

which we use to replace *R3* in (8)

$$q_s \propto \frac{q^2 S^2}{D^{3/2}} \quad (13)$$

This can be written in an interesting form

$$\frac{q_s}{q} \propto \frac{qS^2}{D^{3/2}} \quad (14$$

which we will discuss in class. For our purposes, the really useful to look at (13) as

$$S \propto \frac{\sqrt{q_s} D^{3/4}}{q} \quad (14)$$

This simple relation provides a very useful back-of-the-envelope tool to assist in the analysis of potential channel change. In essence, it provides a function that represents the Lane Balance. We will use it to discuss some different examples of channel change.

And, finally …

An interesting problem we will discuss in class is that in which the sediment supply Q_s increases, but also becomes finer. Guidance from the Lane balance is ambiguous, but this is clearly an important problem in cases where various activities (forest fire, agricultural clearance, urban development, reservoir flushing, dam removal) have the potential of introducing a large amount of fine sediment into a coarse-bedded stream.